



American
Mathematics Olympiad

SECONDARY 3 (GRADE 9) MOCK TEST PAPER SET 1

NAME: _____

Index Number:

SCHOOL: _____

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INSTRUCTIONS:

1. Please **DO NOT OPEN** the contest booklet until the Proctor has given permission to start
2. TIME: **1 hour 30 minutes**.
3. There are 25 questions with 100 total points:
Section A: Questions 1 to 15 score 3 points each, no points are deducted for an unanswered or wrong answer.
Section B: Questions 16 to 20 score 5 points each, no points are deducted for an unanswered or wrong answer.
Section C: Questions 21 to 25 score 6 points each, no points are deducted for an unanswered or wrong answer.
4. Shade your answers neatly using a **2B lead pencil** in the Answer Entry Sheet.
5. PROCTORING: No one may help any student in any way during the contest.
6. No electronic devices capable of storing and displaying visual information are allowed during the course of the exam. Strictly **No Calculators** are allowed.
7. All students must fill and shade their **Name, School and Index Number** in the Answer Entry Sheet and Contest booklet.
8. MINIMUM TIME: Students must stay in the exam hall for at least 1 hour.
9. A student must show detailed working and transfer answers to the Answer Entry Sheet.
10. **No exam papers and written notes can be taken out by any contestant.**



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Solutions

ROUGH WORKING

Section A: 15 Multiple Choice Questions

3 points each

Question 1

Rationalise the denominator of the following fraction.

$$\frac{1}{\sqrt{6} - \sqrt{3} + \sqrt{2} + 1}$$

- A. $\frac{\sqrt{6} + \sqrt{3} - \sqrt{2} + 1}{2}$
- B. $\frac{\sqrt{6} - \sqrt{3} + \sqrt{2} + 1}{2}$
- C. $\sqrt{6} - \sqrt{3} + \sqrt{2} + 1$
- D. $\sqrt{6} + \sqrt{3} - \sqrt{2} + 1$
- E. $\sqrt{6} - \sqrt{3} - \sqrt{2} + 1$

Question 2

If adding 1 to the numerator and adding 5 to the denominator of a fraction is the same as subtracting 5 from the numerator and subtracting 1 from the denominator of the same fraction, then find the fraction if its decimal expansion is 1.5.

- A. $\frac{6}{5}$
- B. $\frac{12}{8}$
- C. $\frac{4}{2}$
- D. $\frac{12}{6}$
- E. $\frac{10}{8}$

Question 3

Two vectors in the 3-D Euclidian space are given

$$4\hat{i} + 9\hat{j} - 13\hat{k} \text{ and } 7\hat{i} + 6\hat{k} + 5\hat{j}$$

Find the dot product of the 2 vectors.

- A. 17
- B. 147
- C. 5
- D. 151
- E. None of these

Question 4

Find $MN - M - N$ in the rational equation below.

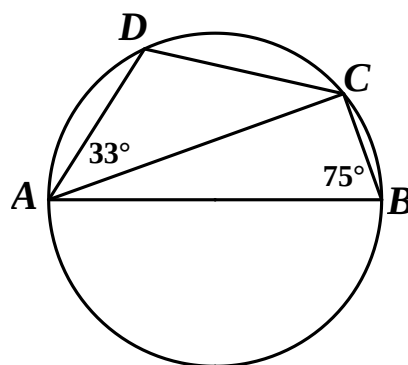
$$\frac{52x + 73}{14x^2 + 83x + 33} = \frac{M}{7x + 3} + \frac{N}{2x + 11}$$

- A. 19
- B. 13
- C. 16
- D. -22
- E. 7

Question 5

In the adjoining figure, AB is the diameter of the circle and C, D lie on the same side of the diameter. If $\angle ABC = 75^\circ$ and $\angle DAC = 33^\circ$, then find the measurement of $\angle DCA$, in degrees.

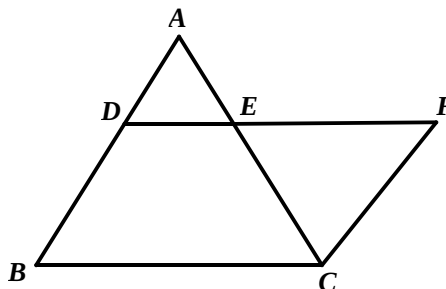
- A. 33
- B. 24
- C. 72
- D. 42
- E. 27



Question 6

In the adjoining figure $\triangle ABC$, point D is on AB such that $BD = 3AD$, E is a point on AC such that $DE \parallel BC$, F is a point on the line DE such that $AB \parallel CF$. Find the length of EF if $BC = 16$.

- A. 12
- B. $\frac{32}{3}$
- C. 4
- D. 8
- E. Not uniquely determined



Question 7

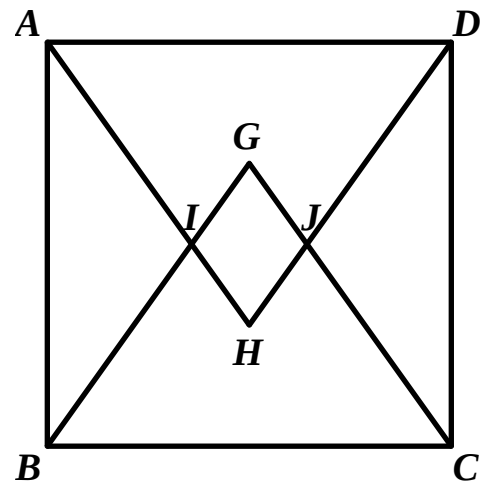
A tangential quadrilateral $ABCD$ is a convex quadrilateral whose sides are tangent to a circle. Given that $AD = 16$, $BC = 8$ and the radius of the circle is 4, find the area of the quadrilateral.

- A. 48
- B. 24
- C. 144
- D. 96
- E. 192

Question 8

In the diagram, $ABCD$ is a square, AHD and BCG are equilateral triangles. Points I and J are intersection points of the sides of the triangles as shown. Given the length of a side of the square is 6, find the area of the rhombus $IGJH$.

- A. $24\sqrt{3} - 36$
- B. 6
- C. $48 - 24\sqrt{3}$
- D. 12
- E. $3\sqrt{3}$



Question 9

What is the greatest number of regions can 7 lines divide a circle into?

- A. 14
- B. 28
- C. 29
- D. 15
- E. 35

Question 10

If $h(x) = 2 + \sqrt{x}$ and $f(h(x)) = 7 + 5\sqrt{x} + x$ then find $f(3)$

- A. 13
- B. $10 + 5\sqrt{3}$
- C. 8
- D. 10
- E. 12

Question 11

In a sequence of numbers, each subsequent term is the sum of cubes of digits of the previous term. If one such sequence starts with the number 457, find the 2021st term in it.

- A. 352
- B. 160
- C. 217
- D. 153
- E. 371

Question 12

The first 5 terms of a sequence are:

5, 9, 16, 28, 47

Find the sum of the 8th and 7th terms

- A. 189
- B. 210
- C. 280
- D. 241
- E. 166

Question 13

How many unique necklaces can be made such that they contain 7 equally spaced beads of 7 different colours?

- A. 5040
- B. 720
- C. 2520
- D. 360
- E. 840

Question 14

Find the value of:

$$\sum_{j=2}^{\infty} \left(\sum_{i=1}^{\infty} \frac{1}{9^i} \right)^{\frac{j}{3}}$$

- A. 1
- B. $\sqrt[3]{\frac{1}{7}}$
- C. $\frac{1}{2} \times \sqrt[3]{\frac{1}{7}}$
- D. $\frac{1}{2}$
- E. $\frac{1}{7}$

Question 15

Given $f: R \rightarrow R$ is a quadratic polynomial

$$f(1) = 1, f(2) = \frac{1}{2}, f(3) = \frac{1}{3}$$

Find $f(4)$

- A. $\frac{1}{4}$
- B. 4
- C. $\frac{7}{24}$
- D. $\frac{1}{2}$
- E. Insufficient information

Section B: 5 Open-Ended Questions**5 points each****Question 16**

For positive real numbers x, y and z , it is given that

$$\frac{x}{3y + 2z} + \frac{3y}{x + 2z} + \frac{2z}{x + 3y} = \frac{3}{2}.$$

Find the value of

$$\frac{(7x + 12y)^2}{z^2}.$$

Question 17

How many times the digit '0' appears in the list of numbers from 1000 to 2021 including both numbers?

Question 18

If $f(x) = 2^x + 86$ and $g(x) = 3x^2 + x - 4$ then find:

$$g[f^{-1}(g(14))]$$

Question 19

What is the shortest distance an ant needs to travel from its current position (7,10) to the anthill (19,25) if the ant must touch the x-axis where sugar is spread all along with it?

Question 20

A number in base 3 has all its digits as 1, but its last digit is 0 in base 2. Which is the largest 4-digit number, in base 10, that has this property?

Section C: 5 Open-Ended Questions

6 Points Each

Question 21

If the following is given:

$$x^4 + x^3 + x^2 + x + 1 = 0$$

Then find the value of $x^{2021} - x$

Question 22

Find the sum of three positive integers p, q, r , if the following is given:

$$\gcd(p, q) = 45, \quad \gcd(q, r) = 21, \quad \gcd(p, r) = 3$$

$$\text{lcm}(p, q) = 4725, \quad \text{lcm}(q, r) = 11025, \quad \text{lcm}(p, r) = 6615$$

Question 23

Find the ordered set of 4-tuples, of positive integers, $[w, x, y, z]$ such that:

$$w + x + y + z = 21$$

Question 24

Find the value of a positive integer x .

$$x = \sqrt{5 + \sqrt{13 + \sqrt{5 + \sqrt{13 + \dots}}}}$$

Question 25

Seven unique awards need to be handed to 7 exceptional students. If the probability that the chief guest hands all awards to the incorrect recipient is $\frac{a}{b}$ in simplest form, then find the value of $a + b$. (It is given that the probability to do so with 5 and 6 awards is $\frac{11}{30}$ and $\frac{53}{144}$, respectively.)

ANSWER KEY

QUESTION 1	A
QUESTION 2	B
QUESTION 3	E
QUESTION 4	A
QUESTION 5	D
QUESTION 6	A
QUESTION 7	D
QUESTION 8	A
QUESTION 9	C
QUESTION 10	A
QUESTION 11	A
QUESTION 12	C
QUESTION 13	D

QUESTION 14	D
QUESTION 15	D
QUESTION 16	0484
QUESTION 17	0335
QUESTION 18	0248
QUESTION 19	0037
QUESTION 20	03280
QUESTION 21	0000
QUESTION 22	1857
QUESTION 23	1140
QUESTION 24	0003
QUESTION 25	0383