



American
Mathematics Olympiad

**SECONDARY 4/ JUNIOR COLLEGE 1 & 2
(GRADE 10,11,12) MOCK TEST PAPER SET 1**

NAME: _____

Index Number:

SCHOOL: _____

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INSTRUCTIONS:

1. Please **DO NOT OPEN** the contest booklet until the Proctor has given permission to start
2. TIME: **1 hour 30 minutes**.
3. There are 25 questions with 100 total points:
Section A: Questions 1 to 15 score 3 points each, no points are deducted for an unanswered or wrong answer.
Section B: Questions 16 to 20 score 5 points each, no points are deducted for an unanswered or wrong answer.
Section C: Questions 21 to 25 score 6 points each, no points are deducted for an unanswered or wrong answer.
4. Shade your answers neatly using a **2B lead pencil** in the Answer Entry Sheet.
5. PROCTORING: No one may help any student in any way during the contest.
6. No electronic devices capable of storing and displaying visual information are allowed during the course of the exam. Strictly **No Calculators** are allowed.
7. All students must fill and shade their **Name, School and Index Number** in the Answer Entry Sheet and Contest booklet.
8. MINIMUM TIME: Students must stay in the exam hall for at least 1 hour.
9. A student must show detailed working and transfer answers to the Answer Entry Sheet.
10. **No exam papers and written notes can be taken out by any contestant.**



SIU SOUTHERN ILLINOIS UNIVERSITY
CARBONDALE STEM EDUCATION
RESEARCH CENTER

Section A: 15 Multiple Choice Questions

3 points each

Question 1

Find the sum of all the prime factors of N if:

$$\log_3 \log_5 \left(\frac{N}{2} \right) = 6$$

- A. 6
- B. 5
- C. 7
- D. 3
- E. 2

Question 2

If $N = 5! 8! 9!$ then find the number of factors of N which are square integers.

- A. 144
- B. 24
- C. 72
- D. 128
- E. 180

Question 3

Find the value of:

$$X = \cos(57) \cos(27) + \sin(57) \sin(27)$$

If $\cos(57) = 0.5446$, $\cos(27) = 0.8910$ and all values in side sin and cos are in degrees

- A. 0.1045
- B. 0.5
- C. $\frac{\sqrt{3}}{2}$
- D. $\frac{1}{\sqrt{2}}$
- E. 1

Question 4

If $x + \frac{1}{x} = \sqrt{3}$ then find the value of $x^3 + \frac{1}{x^3}$:

- A. 1
- B. $3\sqrt{3}$
- C. $-\sqrt{3}$
- D. 0
- E. 3

Question 5

Find the following sum if $i = \sqrt{-1}$:

$$S = i + 2i^2 + 3i^3 + \dots + 2020i^{2020} + 2021i^{2021}$$

- A. $1011 + 1010i$
- B. $1010 + 1011i$
- C. $1010 - 1010i$
- D. $2020 + 2021i$
- E. $-1011 - 1011i$

Question 6

If $a > b > 1$ and $\frac{1}{\log_a b} + \frac{1}{\log_b a} = \sqrt{293}$, then find the value of:

$$\frac{1}{\log_{ab} b} - \frac{1}{\log_{ab} a}$$

- A. 18
- B. 17
- C. 1
- D. 29
- E. 19

Question 7

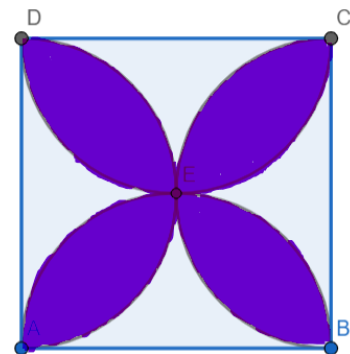
A tower 40m tall, has an elevation of 45 degrees from a point B on the ground, and 53 degrees from a point C on the ground. If point A lies on the base of the tower 40m tall, then what is the distance between point B and C on the ground if $\angle BAC = 90^\circ$ (Given $\sin(37) = \frac{3}{4}$).

- A. 50m
- B. 40m
- C. $40\sqrt{2}$ m
- D. $25\sqrt{3}$ m
- E. 37m

Question 8

In the adjoining figure, 4 semi-circles are constructed and the area of overlap between any 2 semi-circles is shaded purple. Find the area of the shaded region if the side of the square is 10.

- A. $50\pi - 50$
- B. $100 - 25\pi$
- C. $25\pi - 50$
- D. $50\pi - 100$
- E. $100\pi - 100$



Question 10

A quadratic equation of the form $f(x) = x^2 + bx + c$ opens upwards and has its vertex 5 units above the x-axis and 7 units to the right of the y-axis. If a linear equation $g(x) = mx + 10$ were added to the original equation to give $h(x) = f(x) + g(x)$. Find the x coordinate of the vertex if the graph of $h(x)$ just touches the positive x axis.

- A. 7
- B. 5
- C. 6
- D. 10
- E. 8

Question 11

In a class of 40 students, 20 boys and 20 girls, when given a choice to play either football or chess 5 girls and 12 boys preferred football with the rest choosing chess.

What is the probability that the winner of the chess tournament held between the students who choose chess is a girl if all students are equally likely to win?

- A. $\frac{3}{8}$
- B. $\frac{1}{2}$
- C. $\frac{15}{23}$
- D. $\frac{12}{17}$
- E. $\frac{4}{5}$

Question 12

If $\frac{1}{x+1} + \frac{12}{y+12} + \frac{144}{z+144} = 1$, then find the value of $\frac{x^2}{x^2+x} + \frac{y^2}{y^2+12y} + \frac{z^2}{z^2+144z}$.

- A. 3
- B. 12
- C. 157
- D. 2
- E. 11

Question 13

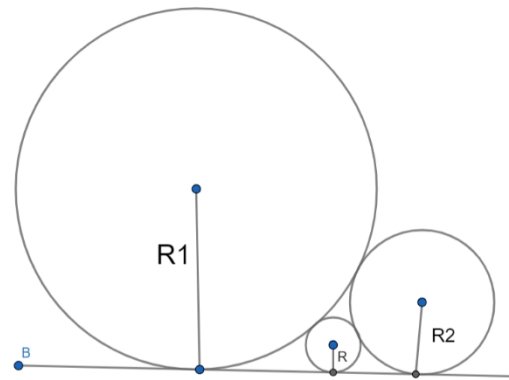
Find the product of all integers $n \geq 1$ such that $\frac{n^3+9}{n^2+13}$ is an integer:

- A. 2
- B. 13
- C. 65
- D. 22
- E. 26

Question 14

In the adjoining figure, if value of the larger 2 circles is 72 and 18 then find the radius of the smallest circle. It is known that all circles are tangent to each other and the common ray B.

- A. 4.5
- B. 14.4
- C. 8
- D. 6
- E. 7.2



Question 15

A person invests \$100 every month in a Recurring deposit which compounds monthly promising a 10% increase, in the principal invested, every year. Find the total value to the recurring deposit after 4 years or after the 48th instalment is paid.

(Given $1.1^{\frac{1}{12}} = 1.00797$)

- A. \$582.3
- B. \$766
- C. \$7660
- D. \$5823
- E. \$4800

Section B: 5 Open-Ended Questions

5 points each

Question 16

If $a, b, c \in R$, $a + b + c = 36$, $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 39$ find:

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} + \frac{a}{c} + \frac{c}{b} + \frac{b}{a}$$

Question 17

Find abc if a, b, c are distinct prime numbers, and the following is given:

$$a + b + c = 92 \text{ and } ab + bc + ca = 2201$$

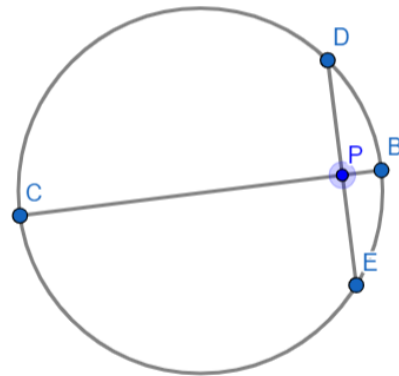
Question 18

If a and b are relatively prime positive integers, then find the value of a + b if:

$$\frac{a^3 - b^3}{(a - b)^3} = \frac{133}{3}$$

Question 19

In the adjoining figure, line segments DE is constructed perpendicular to the diameter BC of the circle intersecting BC at point P. Find the length of the line segment if the radius of the circle is 10 and PB = 2.



Question 20

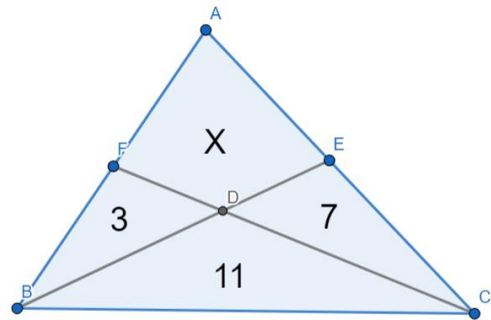
10 lines are drawn on a plane. Find the maximum number of unique line segments that can be identified from these 10 lines on the plane. Note: the line segment identified must be between 2 points that already lies on one of the 10 lines and AB is equivalent to BA.

Section C: 5 Open-Ended Questions

6 points each

Question 21

In the adjoining figure area of the regions are mentioned in the figure (not to scale). Find the value of $m + n$, if $\frac{m}{n}$ is the unknown area x , of the quadrilateral, in the lowest terms.



Question 22

Find the maximum value of the following expression.

$$\sqrt{781 - X} + \sqrt{X - 59}$$

Question 23

Find the ordered set of 4-tuples, of positive integers, $[w, x, y, z]$ such that:

$$w + x + y + z = 21$$

Question 24

If the following is true:

$$\frac{3 + \cot 78^\circ \times \cot 18^\circ}{\cot 78^\circ + \cot 18^\circ} = \frac{\tan n^\circ + \sin m^\circ}{\cos m^\circ}$$

Where n and m are in degrees and are acute angles, find the product mn .

Question 25

Find the value of:

$$X = \sum_{n=1}^7 \tan^2 \frac{n\pi}{16}$$

ANSWER KEY

QUESTION 1	C
QUESTION 2	A
QUESTION 3	C
QUESTION 4	D
QUESTION 5	B
QUESTION 6	B
QUESTION 7	A
QUESTION 8	D
QUESTION 9	A
QUESTION 10	E
QUESTION 11	C
QUESTION 12	D
QUESTION 13	D

QUESTION 14	C
QUESTION 15	D
QUESTION 16	1401
QUESTION 17	4042
QUESTION 18	0023
QUESTION 19	0012
QUESTION 20	0360
QUESTION 21	0193
QUESTION 22	0038
QUESTION 23	1140
QUESTION 24	0270
QUESTION 25	0035